Negative / Rational Exponents: When working on a problem, the **negative exponents need to be made positive.** To change negative exponents into positive exponents, write the reciprocal of the base and now write the exponent as positive. "**If the negative exponent is on top, move it down; if the negative exponent is on the bottom; move it up.:**" If the exponent is negative AND rational – first make exponent positive. Then evaluate rational exponent by writing a radical (**denominator = index; numerator = power**).

Examples:

x^{-2} ← Get the reciprocal, or move the negative exponent down.

$$x^{-2} = \frac{x^{-2}}{1} = \frac{1}{x^2}$$

$5y^{-3}$ ← Get the reciprocal of only the base with the negative exponent, the number stays in its place.

$$5y^{-3} = \frac{5y^{-3}}{1} = \frac{5}{y^3}$$

$\frac{a^2}{b^{-4}}$ ← Get the reciprocal of the base with the negative exponent, the base with the positive exponent stays in its place.

$$\frac{a^2}{b^{-4}} = \frac{a^2 \cdot b^4}{1} = a^2 \cdot b^4$$

$5^{-3} = \frac{1}{5^3}$ ←Write 5 under 1 and write exponent as positive. Evaluate.

$= \frac{1}{125}$

$27^{\frac{-4}{3}}$

$= (\frac{1}{27})^{\frac{4}{3}}$ ← write reciprocal of base; write exponent as positive

$= (\sqrt[3]{\frac{1}{27}})^4$ ←Change to radical. Denominator is index. Numerator is power.

$= (\frac{1}{3})^4 = \frac{1}{81}$ ←Evaluate radical (cube root of 1 and of 27) then evaluate power (1^4 and 3^4)

Simplify by writing with positive exponent. Then simplify completely /find the value. Write final answer with positive exponents in variables.

1) 8^{-1} $= \frac{1}{8^1} = \frac{1}{8}$

2) 3^{-2} $= \frac{1}{3^2} = \frac{1}{9}$

5) $(3x)^{-1}$ $= \frac{1}{(3x)^1} = \frac{1}{3x}$

7) $4c^{-3}$ $= 4 \cdot \frac{1}{c^3} = \frac{4}{c^3}$

8) $2pr^{-5}$ $= 2p \cdot \frac{1}{r^5} = \frac{2p}{r^5}$

12) $\frac{5}{z^{-3}} = 5z^3$ reciprocal is z^3

13) $-\frac{2x}{a^{-4}} = -2a^4x$

14) $\frac{3b}{-5c^{-1}} = -3bc$

21) $\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$

22) $\left(\frac{2}{5}\right)^{-2} = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

23) $\left(\frac{2a}{9c}\right)^{-2} = \left(\frac{9c}{2a}\right)^2 = \frac{81c^2}{4a^2}$

24) $125^{-2/3} = \frac{1}{125^{2/3}} = \frac{1}{(\sqrt[3]{125})^2} = \frac{1}{5^2} = \frac{1}{25}$

25) $\left(\frac{9}{16}\right)^{-1/2} = \left(\frac{16}{9}\right)^{1/2} = \sqrt{\frac{16}{9}} = \frac{4}{3}$

Answers:

1) $\frac{1}{8}$ 2) $\frac{1}{9}$ 5) $\frac{1}{3x}$ 7) $\frac{4}{c^3}$ 8) $\frac{2p}{r^5}$ 12) $5z^3$ 13) $-2xa^4$

14) $-\frac{3bc}{5}$ 21) $\frac{4}{3}$ 22) $\frac{25}{4}$ 23) $\frac{81c^2}{4a^2}$ 24) $\frac{1}{25}$ 25) $\frac{4}{3}$

Step	Method	Example
1	Label all unlabeled exponents "1"	$\left(\frac{25x^{-6}y^{1}(z^{-11})^{2}}{5(x^{-2})^{5}y^{8}z^{2}}\right)^{-2}$
2	**Get rid of any inside parentheses** by multiplying the exponents.	$\left(\frac{25x^{-6}y^{1}z^{-22}}{5x^{-10}y^{8}z^{2}}\right)^{-2}$
3	**Combine all like bases** (*coefficients/ same letters*). -Divide coefficient/reduce fraction/leave as fraction. -Exponent law for variables: Top exponent minus bottom exponent, *if dividing*. (If bottom exponent is negative, remember –(-) = +.	$\left(5x^{4}y^{-7}z^{-24}\right)^{-2}$
4	**Distribute outside exponents to all terms inside bracket.** Raise the coefficient to the exponent. Multiply the exponent by the variables' exponents.	$5^{-2}x^{-8}y^{14}z^{48}$
5	Move all negatives either up or down. Make the exponents positive.	$\frac{y^{14}z^{48}}{5^{2}x^{8}}$
6	Find the value of the coefficient raised to an exponent.	$\frac{y^{14}z^{48}}{25x^{8}}$

Now try this one : $\left(\frac{18x^{-4}(y^{3})^{-2}(z^{4})^{2}}{10(x^{2})^{-3}y^{4}z^{-2}}\right)^{-2}$ $= \left(\frac{18x^{-4}y^{-6}z^{8}}{10x^{-6}y^{4}z^{-2}}\right)^{-2} = \left(\frac{9x^{2}y^{-10}}{5}\right)$

$= \left(\frac{9}{5}\right)^{-2}x^{-4}y^{20}z^{-20}$

$= \left(\frac{5}{9}\right)^{2}x^{-4}y^{20}z^{-20}$

$= \frac{25y^{20}}{81x^{4}z^{20}}$

A. To simplify these, use the index of the radical as the denominator of the exponent. Also find the root of the coefficient, if possible. Or write coefficient as mixed radical.

1. $\sqrt{25a^{18}b^{20}}$ 2. $\sqrt{8x^6y^8}$ 3. $\sqrt{x^{11}}$

4. $\sqrt[5]{\frac{x^5}{y^{10}}}$ 5. $\sqrt[3]{8x^4y^3}$ 6. $\sqrt[4]{81x^5y^2z^8}$

B. True or False?

1. $9^{1/3} = \sqrt[3]{9}$ (T)
2. $8^{5/3} = \sqrt[5]{8^3}$ (F)
3. $(-16)^{1/2} = -16^{1/2}$ (F)
4. $9^{-3/2} = \frac{1}{27}$ (T)
5. $6^{-1/2} = \frac{1}{\sqrt{6}}$ (T)
6. $\frac{2^1}{2^{1/2}} = 2^{1/2}$ (T)
7. $2^{1/2} \cdot 2^{1/2} = 4^{1/2}$ (T)
8. $16^{-1/4} = -2$ (F)
9. $6^{1/6} \cdot 6^{1/6} = 6^{1/3}$ (T)
10. $(2^8)^{3/4} = 2^6$ (T)

Answers

A 1). $5a^9b^{10}$ 2). $2\sqrt{2}x^3y^4$ 3) $x^{\frac{11}{2}}$ 4) $\frac{x}{y^2}$ 5) $2x^2y$ 6) $3x^{\frac{5}{4}}y^{\frac{1}{2}}z^2$

B. 1. T | 2. F | 3. F | 4. T | 5. T | 6. T | 7. T | 8. F | 9. T | 10. T